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Introduction to Finance with MATLAB

University Paris-Dauphine
Magistère BFA & Master 2 EID

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Lecture 5: Bonds and Interest Rates Valuation

Content of the Lecture

Objectives of this lecture:

- Compute the price of a zero coupon bonds;
- Calculate its interest rate;
- Use loops and functions.

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1. Introduction

This section is devoted to exercises based on the valuation of bonds on financial markets.

2. Interest rates and bonds valuation exercises

Exercise 1

Consider a P=100 coupon bond of nominal value (face value) N=100 with time maturity T=5 and annual coupon rate c=0.08.

t	0	1	2	3	4	5
cashflow	-P	cN	cN	cN	cN	cN+N
discounted cashflows	...	...	...	...	...	...

1. After declaring P, N, T and C in MATLAB, define the vector line, denoted cf, in which is reported cash flows inherent to the purchase of the obligation.

2. Determine the internal rate of return (or discount rate) or the rate of return at maturity that cancels the net present value of the income stream. Remember that:
$$P = \sum_{t=1}^{T} \frac{C_t}{(1+\rho)^t} + \frac{N}{(1+\rho)^T}$$
   This is a T-root polynomial which can be solved through the roots(·) function. To do so, employ functions roots(·) and fliplr(·). Tips: compte the roots of the problem, and select the unique real root discarding imaginary ones.

3. Determine the internal rate of return using the command irr(·).

4. Determine the internal rate of return using the non-linear solver fsolve(·): A working example of fsolve(·) for a two period bond: $P = (1+\rho)^{-1}cN + (1+\rho)^{-2}(1+c)N$ is obtained:

```
f=@(x)([
        -P + C/(1+x(1)) + N*(1+c)/(1+x(1))^2
]);
x0=[0];
rho=fsolve(f,x0)
```

   Adapt this code for the exercise.

Exercise 2

Electricité de France SA has capped a $12.4 billion global fundraising in 2014 by selling the first 100-year bonds in Europe with coupon rate of 6.125%.

1. Assuming that each obligation has $P = N = 100$, determine the internal rate of return of this bond.
2. Three years later, the price of bond is now 90. Determine the new rate of return.

Exercise 3

Consider a bond, repaid at maturity with facial value N, maturity n and coupon c. We want to determine the price of this obligation P with the rate of return r. (i) the first method aims at determining the analytical solution of P and code it. (ii) the second solution use a loop parsing the vector of cash flows. (iii) the last one employs the function `pvvar(·)`.

1. Determine the price of the obligation using the three methods, assuming that N=100, c=0.08, r=0.05 and n=5.
2. Adapt the code for each method in order to compute the bond price for different internal rate of return. Construct a vector r=[0.08 0.09 0.10 0.11 0.12] and use this variable internal rate of return.
3. Discuss your result regarding the bond price.

Exercise 4

Consider a zero-coupon bond with price P, nominal value N and maturity n. Letting r denote the variable internal rate of return, the bond price reads as:

$$P = \frac{N}{(1+r)^n}$$

We seek to measure the implication of maturity n over the bond price P following a change in the rate of return r.
We fix $N = 100$ and consider two possible maturities $n = [5, 20]$.

1. Determine the variation of the bond price when r change from 0.08 to 0.09. Contrast your result for the two maturities $n = [5, 20]$.
2. Same exercise, but employ loops to store your calculation of bonds price into a matrix 2x2.

3. Two assets portfolio

Exercise 5

Get this working code:

```
% dl Assets
Asset1 = getMarketDataViaYahoo('GOOG', '01-01-2005', 'now', '1mo');   % google
Asset2 = getMarketDataViaYahoo('AABA', '01-01-2005', 'now', '1mo');   % yahoo
Asset3 = getMarketDataViaYahoo('^GSPC','01-01-2005', 'now', '1mo');   % SP500
Asset4 = getMarketDataViaYahoo('^TNX','01-01-2005', 'now', '1mo');    % 10 year
% matrix of asset prices
P = [ Asset1.AdjClose Asset2.AdjClose Asset3.AdjClose ];
```

In this code, P is a T×N matrix of N assets and T periods (here 3 assets). Asset1 is google share price, Asset2 is yahoo share price, Asset3 is the SP500 and Asset4 is the return from holding a risk free asset in the US (annual basis).

1. From matrix P, compute the return of holding these assets and store it in matrix R.

2. Compute the monthly riskless rate on the same time period. Recall to divide the riskless rate by 12 to approximate the monthly basis.

3. Capital asset market pricing model is usually expressed as:

$$E\left(R_i\right) = R_f + \beta_i \left(E\left(R_m\right) - R_f\right) \tag{1}$$

   where R_i is the i-th asset, R_f is the riskless rate, R_m is the return from the market.

   (a) Compute spreads $R_i - R_f$ and $R_m - R_f$. Here, R_m si the return of the S&P 500.

   (b) Perform a linear regression of Equation 1. What is the estimated value of β_{google} and β_{yahoo}? How much google asset is expected to increase following an increase of market return by 10%?

   (c) Compare your result of β_{google} and β_{yahoo} with the closed form expression $\beta_i = cov\left(R_i, R_m\right)/var\left(R_m\right)$.

4. Let us now work on a two assets portfolio based on the purchase of Google and Yahoo shares. Letting w and $1-w$ the respective weight for each assets in the portfolio, the return and variance are given by:

$$E\left(R_P\right) = wE\left(R_{google}\right) + (1-w)\,E\left(R_{yahoo}\right)$$
$$\sigma_P^2 = w^2\sigma_{google}^2 + (1-w)^2\,\sigma_{yahoo}^2 + 2w\,(1-w)\,cov\left(R_{google}, R_{yahoo}\right)$$

   (a) Create a vector of weights W going from 0 to 1 with step size of 0.01.

   (b) Compute for each weight of W the associated return and variance.

(c) Plot the resulting portfolio (x axis should be the risk measured by the standard deviation of the portfolio while the y axis should be the return).

(d) Suppose that we are very risk-adverse, select the minimum variance portfolio.

(e) How has the changed the weight of google in this portfolio over time? Create a one-year window of return/variance, and store the weight associated to the minimum variance portfolio. Loop it over the sample period. Plot the result.